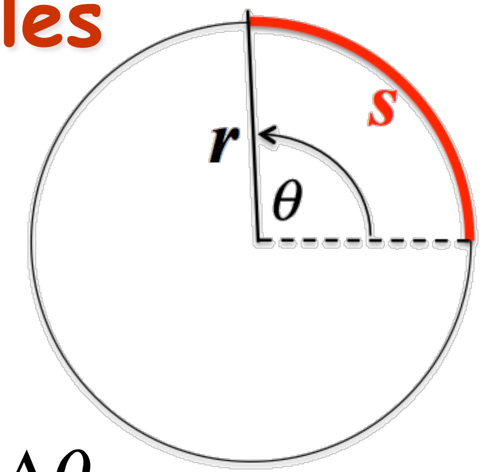
Chapter 10: Rotational Motion
Thursday March 5th

- Review of rotational variables
- Review of rotational kinematics equations
- Rotational kinetic energy
- Rotational inertia
- Rolling motion as rotation and translation
- Torque and Newton's 2nd law (if time)
- Examples, demonstrations and *i*clicker

- Normal schedule after spring break, starting Monday 16th.
- Material covered today relevant to LONCAPA due that day.
- Normal lab schedule after spring break.
- I will return mid-term exams on Tuesday after spring break.

Reading: up to page 169 in Ch. 10

Review of rotational variables



Angular position: $$\theta = \frac{s}{r} \quad (\text{in radians})$$

Angular displacement: $$\Delta\theta = \theta_2 - \theta_1$$

Average angular velocity: $$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

Units: rad.s^{-1}, or s^{-1}

Instantaneous angular velocity: $$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Average angular acceleration: $$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

Units: rad.s^{-2}, or s^{-2}

Instantaneous angular acceleration: $$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

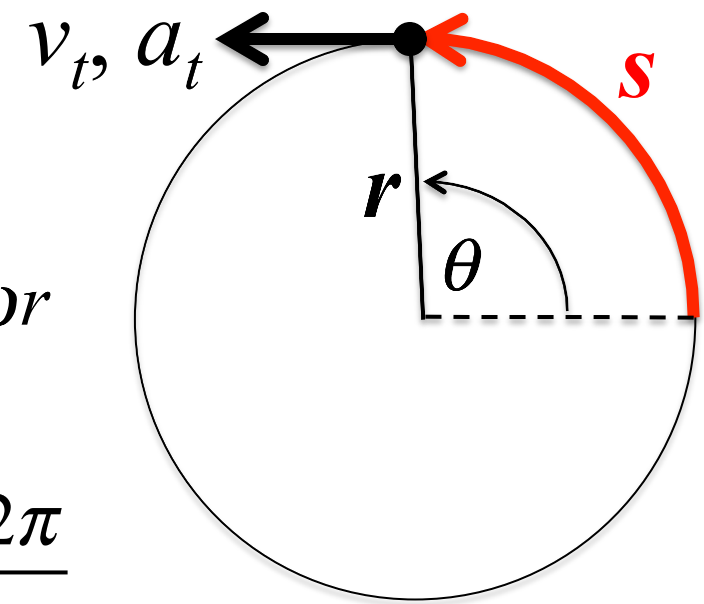
Review of rotational kinematic equations

THE SAME OLD KINEMATIC EQUATIONS

Equation number	Equation	Missing quantity
10.7	$\omega = \omega_0 + \alpha t$	$\theta - \theta_0$
10.8	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$	ω
10.9	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	t
10.6	$\theta - \theta_0 = \bar{\omega} t = \frac{1}{2}(\omega_0 + \omega)t$	α
	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$	ω_0

Important: equations apply ONLY if angular acceleration is constant.

Review: transforming rotational/linear variables



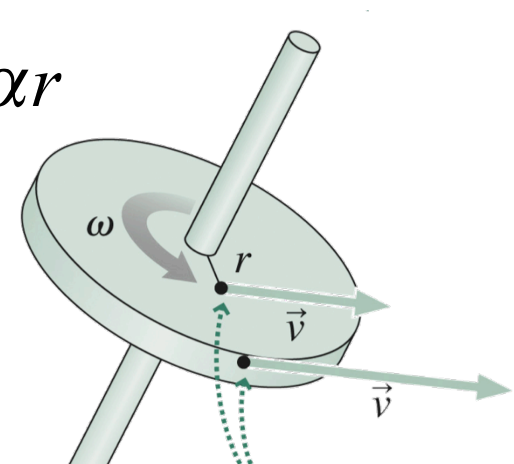
Position: $s = \theta r \ (\theta \text{ in rads})$

Tangential velocity: $v_t = \frac{ds}{dt} = \frac{d\theta}{dt} r = \omega r$

Time period for rotation: $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$

Tangential acceleration: $a_t = \frac{dv_t}{dt} = \frac{d\omega}{dt} r = \alpha r$

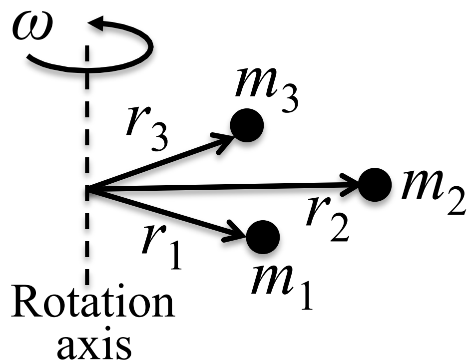
Centripetal acceleration: $a_r = \frac{v^2}{r} = \omega^2 r$



v depends on r

Kinetic energy of rotation

Consider a (rigid) system of rotating masses (same ω):



$$K = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + \tfrac{1}{2}m_3v_3^2 + \cdots\cdots$$

$$= \sum \tfrac{1}{2}m_iv_i^2$$

where m_i is the mass of the ith particle and v_i is its speed.

Re-writing this:

$$K = \sum \tfrac{1}{2}m_i\left(\omega r_i\right)^2 = \tfrac{1}{2}\left(\sum m_ir_i^2\right)\omega^2$$

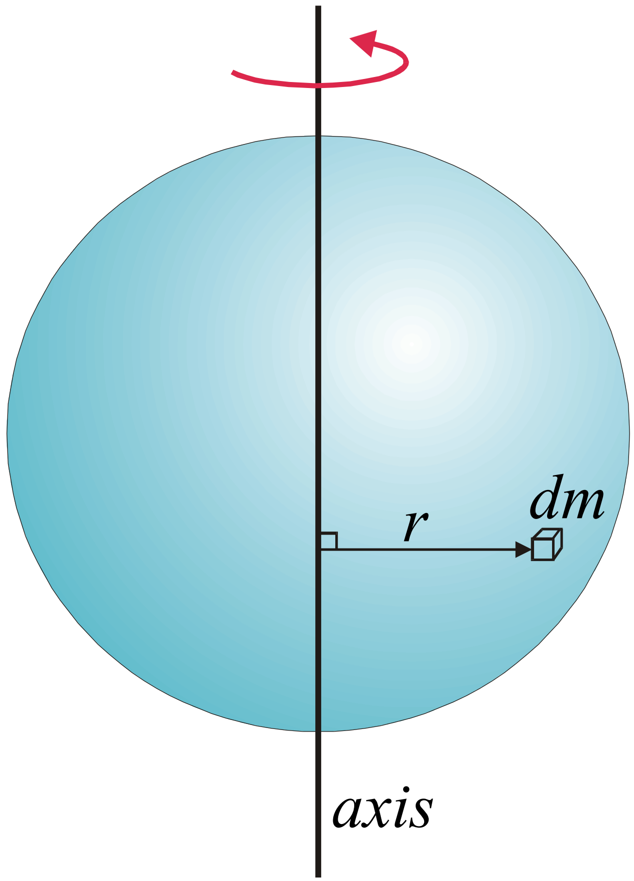
The quantity in parentheses tells us how mass is distributed about the axis of rotation. We call this quantity the rotational inertia (or moment of inertia) I of the body with respect to the axis of rotation.

$$I = \sum m_ir_i^2 \qquad\qquad K = \tfrac{1}{2}I\omega^2$$

Calculating rotational inertia

For a rigid system of discrete objects: $I = \sum m_i r_i^2$

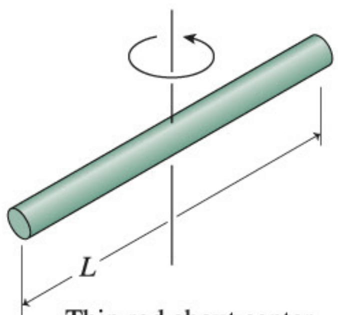
Therefore, for a continuous rigid object: $I = \int r^2\, dm = \int \rho r^2\, dV$



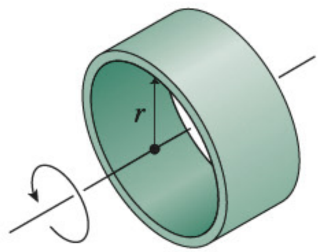
- Finding the moments of inertia for various shapes becomes an exercise in volume integration.
- You will not have to do such calculations.
- However, you will need to know how to calculate the moment of inertia of rigid systems of point masses.
- You will be given the moments of inertia for various shapes.

Rotational Inertia for Various Objects

Table 10.2 Rotational Inertias

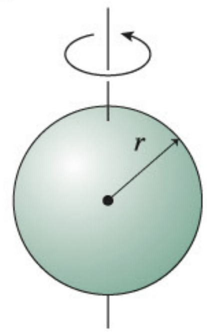


Thin rod about center
$I = \frac{1}{12}ML^2$

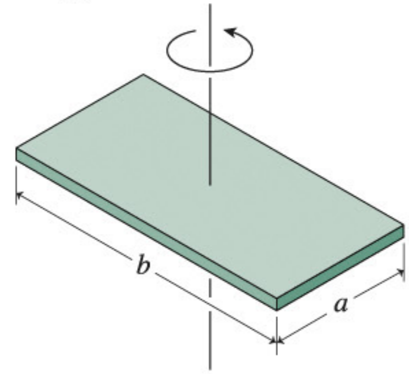


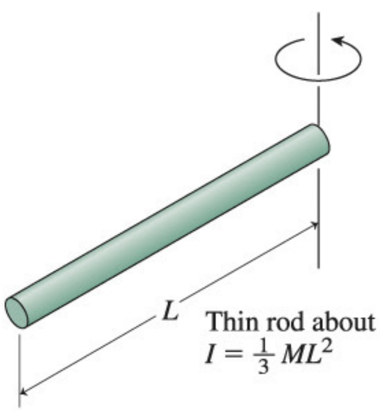
Thin ring or hollow cylinder about its axis
$I = MR^2$

Solid sphere about diameter
$I = \frac{2}{5}MR^2$

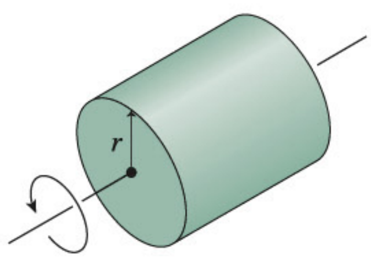


Flat plate about perpendicular axis
$I = \frac{1}{12}M(a^2 + b^2)$



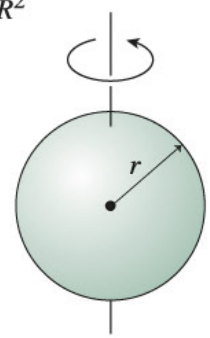


Thin rod about end
$I = \frac{1}{3}ML^2$

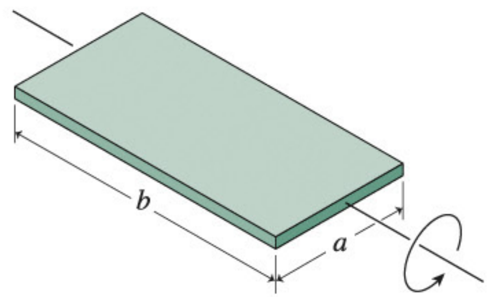


Disk or solid cylinder about its axis
$I = \frac{1}{2}MR^2$

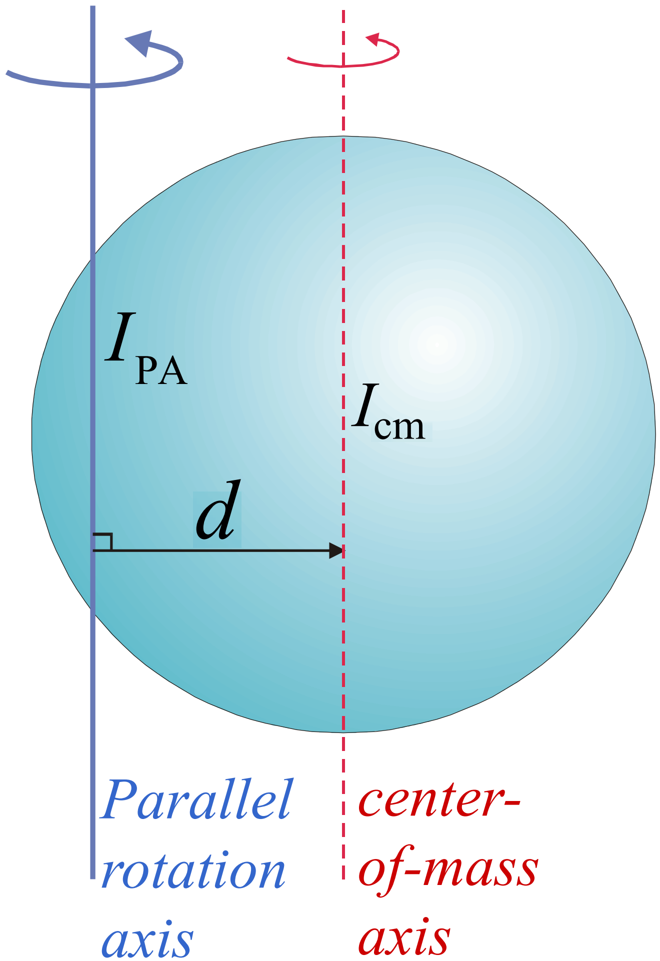
Hollow spherical shell about diameter
$I = \frac{2}{3}MR^2$



Flat plate about central axis
$I = \frac{1}{12}Ma^2$



Parallel axis theorem

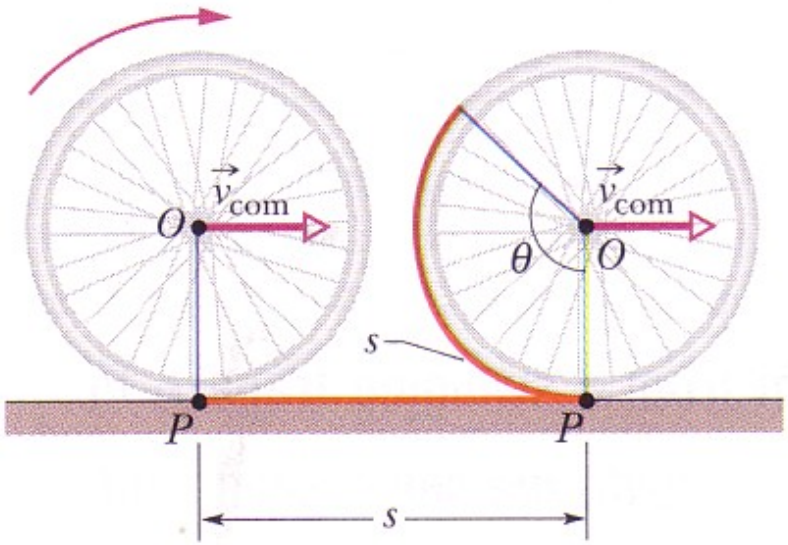


- If you know the moment of inertia of an object about an axis though its center-of-mass (cm), then it is trivial to calculate the moment of inertia of this object about any parallel axis:

$$I_{PA} = I_{cm} + Md^2$$

- Here, I_{cm} is the moment of inertia about an axis through the center-of-mass, and M is the total mass of the rigid object.
- It is essential that these axes are parallel; as you can see from table 10-2, the moments of inertia can be different for different axes.

Rolling motion as rotation and translation

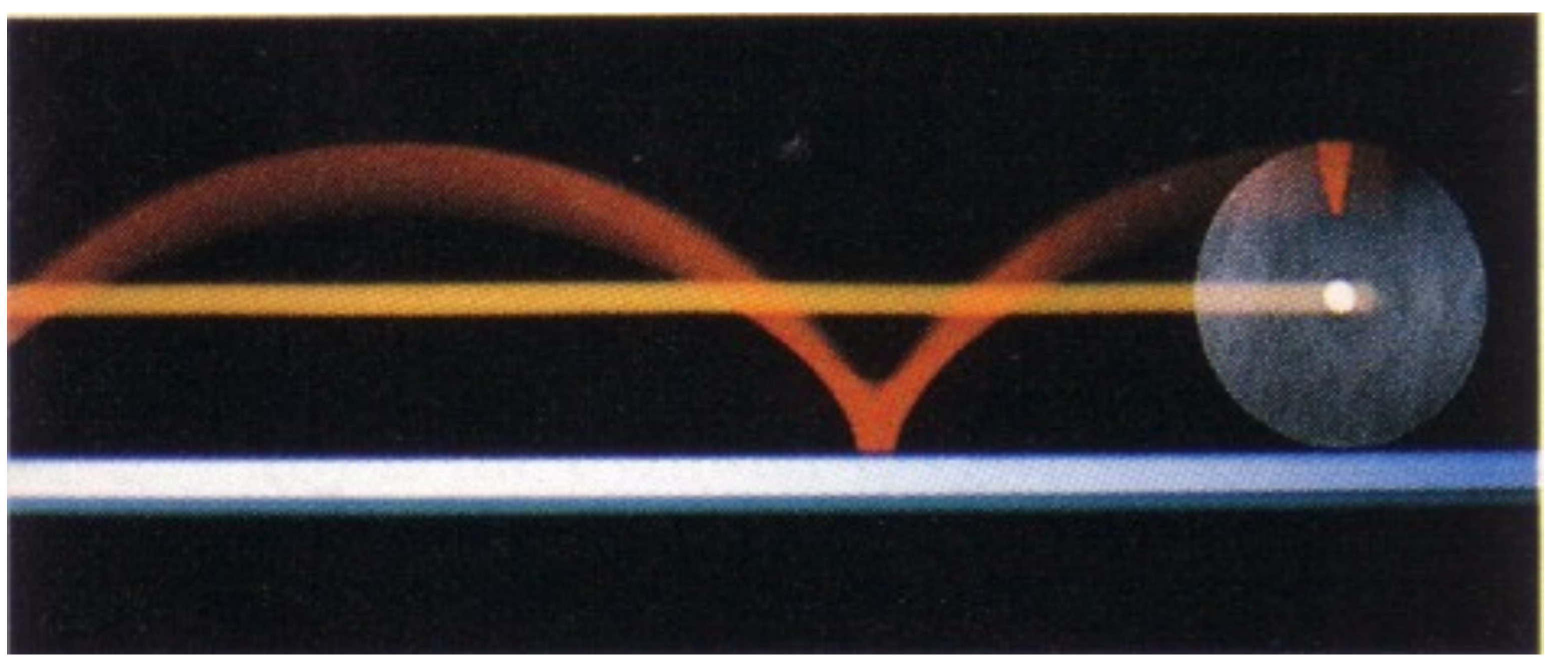


$$s = \theta R$$

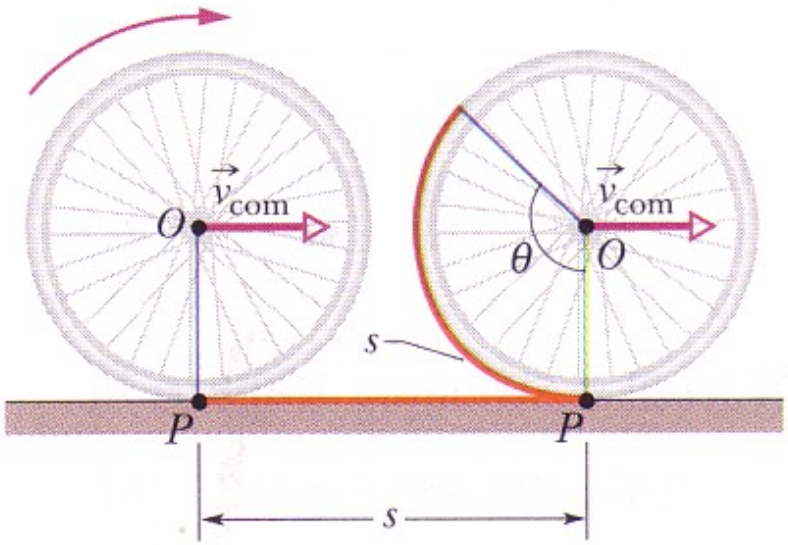
The wheel moves with speed ds/dt

$$\Rightarrow \ v_{cm} = \omega R$$

Another way to visualize the motion:



Rolling motion as rotation and translation

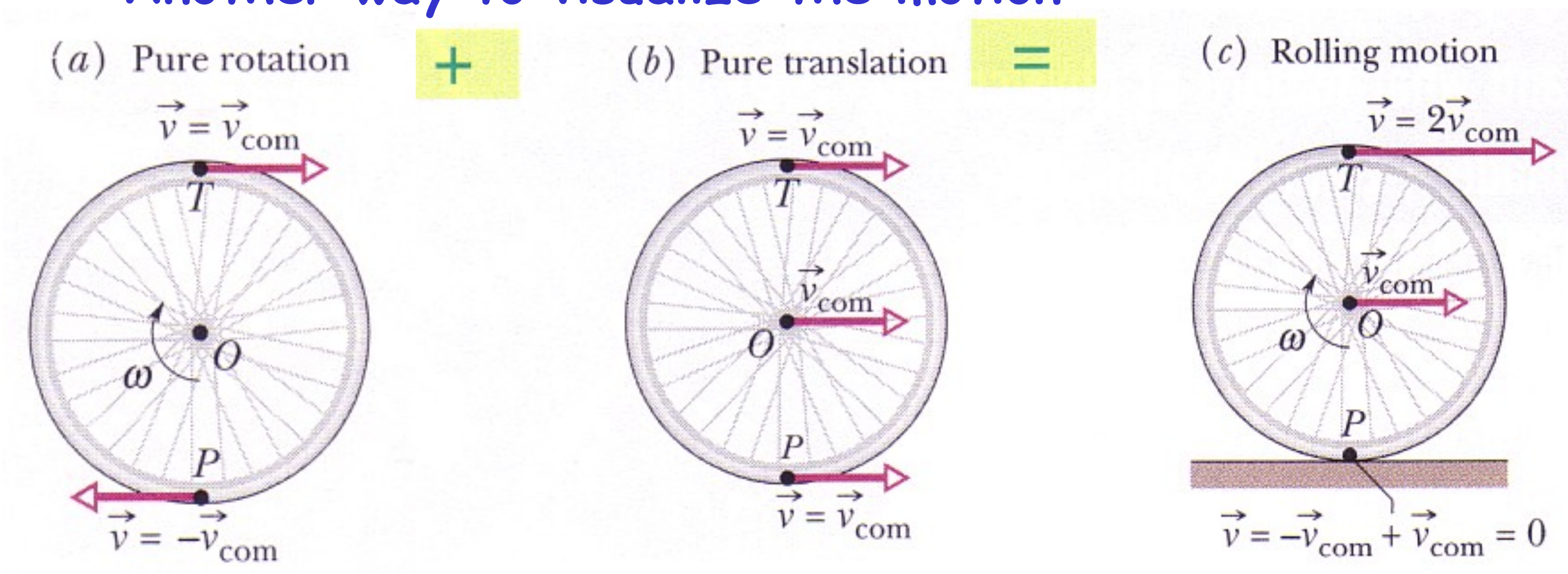


$$s = \theta R$$

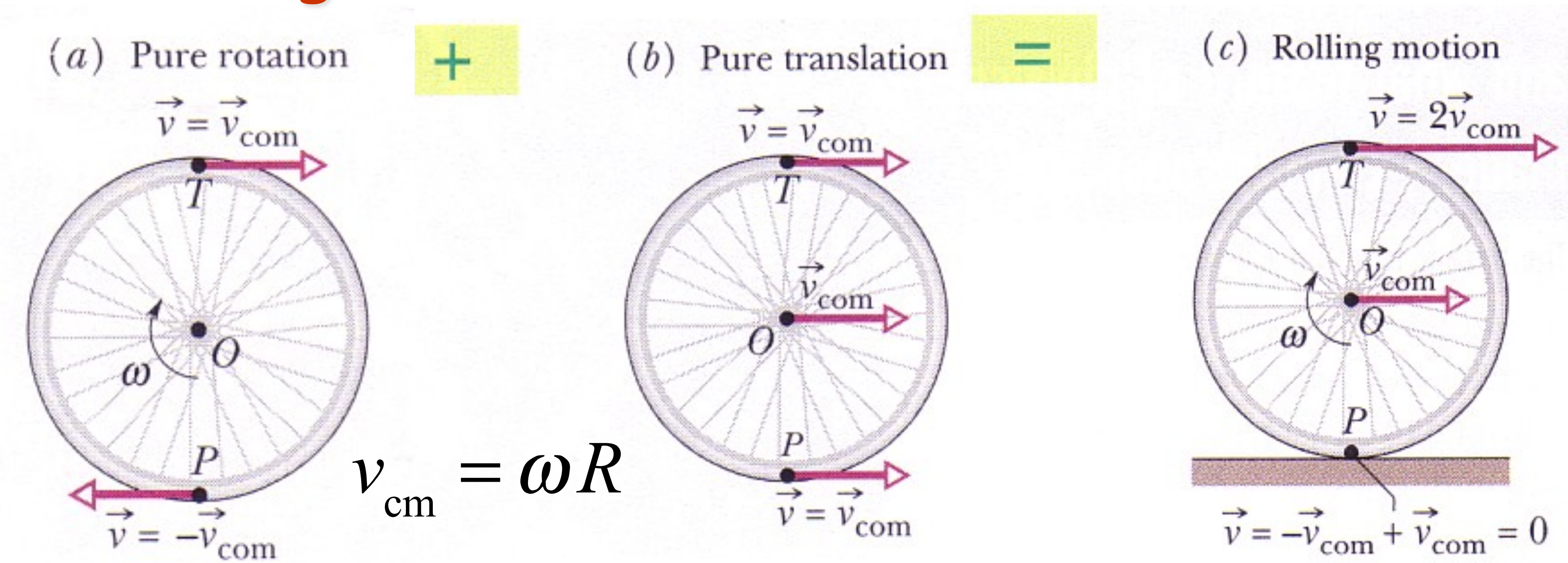
The wheel moves with speed ds/dt

$$\Rightarrow v_{cm} = \omega R$$

Another way to visualize the motion:



Rolling motion as rotation and translation



$$v_{\mathrm{cm}} = \omega R$$

Kinetic energy consists of rotational & translational terms:

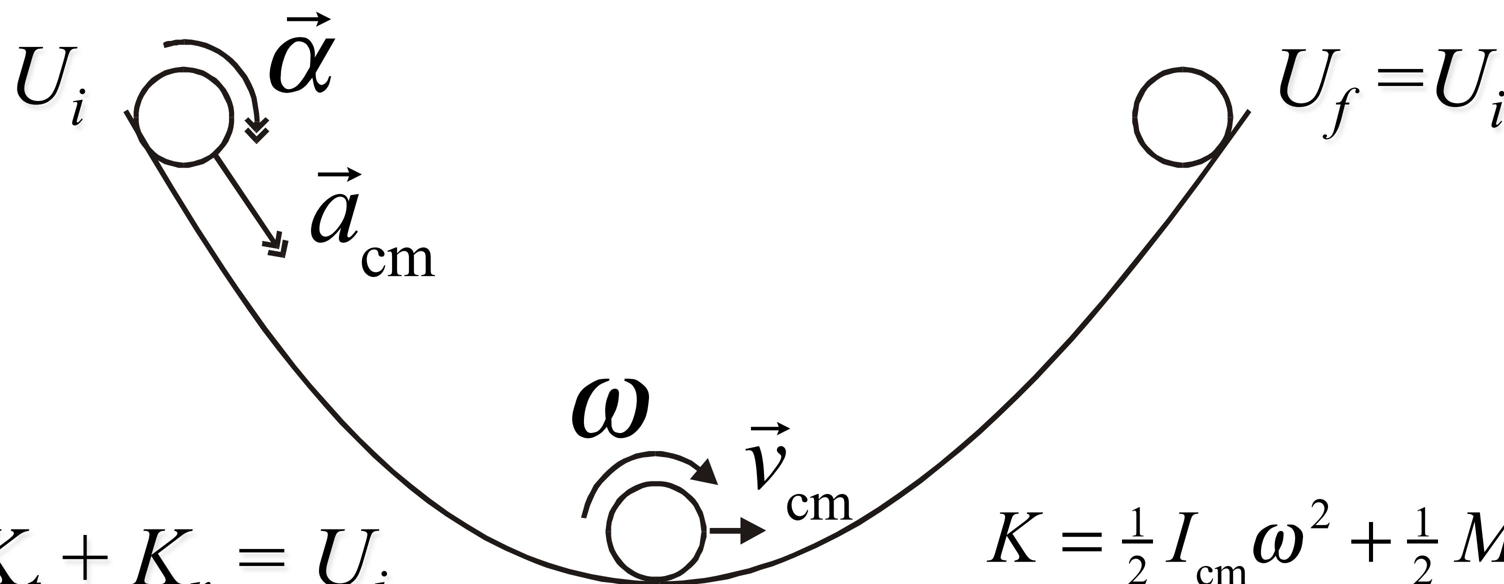
$$K = \tfrac{1}{2} I_{\mathrm{cm}}\omega^2 + \tfrac{1}{2} M v_{\mathrm{cm}}^2 = K_r + K_t$$

$$K = \tfrac{1}{2}\left\{ fMR^2 \right\} \frac{v_{\mathrm{cm}}^2}{R^2} + \tfrac{1}{2} M v_{\mathrm{cm}}^2 = \tfrac{1}{2} M' v_{\mathrm{cm}}^2$$

Modified mass: $M' = \left(1 + f\right) M$ **(look up *f* in Table 10.2)**

Rolling Motion, Friction, & Conservation of Energy

- Friction plays a crucial role in rolling motion (more on this later):
  - without friction a ball would simply slide without rotating;
  - Thus, friction is a necessary ingredient.
- However, if an object rolls without slipping, mechanical energy is <u>**NOT**</u> lost as a result of frictional forces, which do <u>**NO**</u> work.
  - An object must slide/skid for the friction to do work.
- Thus, if a ball rolls down a slope, the potential energy is converted to translational and rotational kinetic energy.



$$K = K_t + K_r = U_i$$

$$K = \tfrac{1}{2} I_{cm}\omega^2 + \tfrac{1}{2} M v_{cm}^2$$